Reason's Nature— The Role of Mathematics

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1 Prelude

Let us start with a little story playing in St. Petersburg in the 18th century. Leonhard Euler (1707-1783), one of the greatest mathematicians of all times, met Denis Diderot (1713-1784), a leading man of the French enlightment and a charming defender of Atheism. Euler was introduced to Diderot as a person who found an algebraic proof for the existence of God. Showing a poker face, Euler said: "Monsieur, we have

$$\frac{a+b^n}{n} = x,$$

thus God exists: your turn!" Diderot was totally baffled and did not manage to reply. Everybody laughed at him, and soon afterwards he returned to France.

Indeed, within a theological or philosophical debate you can easily intimidate your opponent by a mathematical argument. Thereby mathematics takes profit from a property which could almost be described as a *coincidence of opposites*: Namely, on the one hand a mathematical formula, calculation or argument is usually incomprehensable for the opponent. On the other hand, however, it promises a complete transparancy—at least in principle. Thus the miserable opponent—searching for insight—struggles mainly with his own ignorance. So he will not be able to contradict the argument—let alone to question whether it is legitimate to use mathematics at all. Of course, then the mathematician has an easy game to play.

Please don't panik; this paper will not terrorise the reader with any mathematical formalism. Far from it! Mathematics will not be a *tool* for any argumentation, but rather the *object* under consideration.

1.1 Reason's Nature—Twofold Understanding

It is not by chance that the expression 'Reason's Nature' in the in the title can be interpreted in a twofold way.

First, we have 'nature' as the object of inquiry seen by (human) reason; here *reason* denotes the type of question we pose, namely that we are interested in describing our object in a 'reasonable' way. In particular we may think about nature

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within the framework of natural science—a framework developed by Galilei, Newton and their successors which today claims to be *the* canonical frame of reference for any question about nature.

Second, however, we might speculate about the 'natur of reason', i.e., try to describe reason itself. Now *nature* denotes the type of question we pose, namely that we look for the essence of reason, not just some arbitrary features of it. For this second question, it is not so easy to find a cannonical frame of reference. We may just mention the long lasting tradition of philosophical reflection about the human mind or the various attempts of a mental introspection (philosophical, theological, religious, mystical etc.), but also approaches to characterise human reason by the works of human culture: arts, religion, sciences etc.

Apparently it is tempting to draw back to our first reference frame. Our 'best' descriptions of nature are presented by natural science—at least this is often claimed to be so. Thus, why not analyzing reason's nature within this scientific framework? It is thus nowadays alltoo common to identify the above sketched two different endeavours; the result is a picture, where reason has emerged during the course of an evolutionary process and *now* it can be found *within* human brains. We thus try to describe reason as a phenomenon within time and space—whatever time and space might be. Anyway, we find various approaches whithin this scientific perspective, e.g., evolutionary biology and socio-biology, neurophysiology, etc. If these go beyond the investigation of special phenomena and try to present an encompassing and consitent theory of human reason we may call this a naturalistic approach.

In my paper I will first observe what happens if the two perspectives are identified in the above naturalistic way. Second, I will follow the first understanding separately, thus ask for the scientific perspective on nature and the role of mathematics for it. Third, I will follow the second understanding and try to observe mathematics as a quite special ability of human reason. Finally, I will again examine possible relations between these two.

2 Ridiculous Circles

2.1 Naturalism

If we try to take naturalism at its words we might end up with the following or a similar argumentation. Human reason, within human brains, is nothing but an electrical process within a network of brain cells. These brain cells and electricity consist in the movement of elementary particles (or is the evolution of an elementary field, if you prefer this picture). But what *is* an elementary particle? It is just a mathematical structure (to be more precise, an irreducible (unitary) representation of the chosen symetry group of space-time). But how do we now, what a mathematical structure *is*? If we refuse to be platonists—which the naturalist should do—we will interpret mathematics as consisting of symbols or marks written down on the black board with chalk. Now the question is: What is chalk? And I will go on in the argumentation by quoting the great mathematician HERMAN WEYL (1885-1955) who stated it 1948 as follows:

> *As a scientist we may be tempted to argue like that: 'As we know, chalk consists of molecules and these in turn are built from (...) elementary particles (...) However, analysing what theoretical physics means by these words, we saw, that these physical objects are dissolved into a mathematical symbolism; the symbols, however, are finally concrete marks written with chalk on a black board. You certainly will notice the ridiculous circle*[2]*.*

I think, *any* attempt to interpret the scientific results with the aim of a consequently naturalistic position—if it does not invisibilise its consequences—will run into a similar 'ridiculous circle'.

2.2 Projections in Science-Religion

The status, however, of many arguments in the so called 'Science-Religion-Dialogue' is not much better. They also start with a 'maximal interpretation' of science—thereby often projecting the author's preferences or spiritual needs onto the scientific 'facts'. Following this way, lowbrow metaphors such as the 'big bang' or purely technical terms such as 'chaos' gain an unjustified and mostly unclear metaphysical meaning. Remark here, that the interpretation of these scientific 'facts' is often quite arbitrary[3]—the very same astronomy, e.g., can lead to an identification of the universe with a "co-creative cosmos" or a "senseless one" (as JACQUES MONOD coined it).

In the end we might obtain the sort of arguments ALAN SOKAL[4] was scoffing at. Under the title: *Transgressing the Boundaries: Towards a Transformative Hermeneutics of Quantum Gravity* he presented a completely nonsensical paper

[2]See H. WEYL: *Wisssenschaft als symbolische Konstruktion des Menschen.* In: "Gesammelte Abhandlungen." Bd. IV, Springer, Berlin 1968, p. 342.

[3]For more details see [Ni99a].

[4]See A. Sokal: *Transgressing the Boundaries: Towards a Transformative Hermeneutics of Quantum Gravity.* Social Text **46/47**, 217-252.

to the renowned Journal Social Text. By quoting the 'important people' using some stylish political catchwords and mixing this up with an incomprehensible fluff of technical slang from theoretical physics he could bluff the editiors, his nonsense passed the referee process and it was published. In fact a naive transgressing disciplinary boundaries calls for this type of mocking. Just to mention the classical example: Recall VOLTAIRE's *Candide* showing that LEIBNIZ' application of the (mathematical) principle of least action to ethical questions leads to an irremediable confusion.

In our times, we have to face an even more serious situation. Science became an extremely filigree network of specialised disciplines with their very own results and standards of argumentation. Not even the most important results within mathematics could be overseen by one person as it was possible for a HILBERT in 1900; let alone the results of natural sciences. At the same time science has given up its competence for presenting a 'world view'. Every specialist has just a very small range of (practical or theoretical) knowledge, but sometimes an even stronger desire for an all-embracing orientation. Since it seems to be out of style to simply follow the doctrines of a church or a philosophical tradition these people apparently are tempted to tinker their personal 'philosophy' based on the 'results' of science they (pretend to) understand.

It is thus missleading to cross diciplinary boundaries without being aware of changes in language and meaning and without control of the effects. Any uncritical interpretation of the *results* of natural science is a quite problematic endeavour. The question on the epistemologic basis is indispensable.

2.3 Methods instead of Results

To me it seems to be more reasonable to analyse the *instruments* of human reason for doing science istead of using these instruments to analyse reason. The focus then lies on the scientific method, and in particular on *mathematics* and *experiment*. Remark that now the subject is quite constant in contrast to the transient and difficult to understand scientific results.

3 Reason's Nature—Nature mathematically described

It is an indisputable fact that modern science heavily relies on mathematics. Just recall IMMANUEL KANT's (1724-1804) famous claim that *in every special natural* *doctrine only so much science is to be met with as mathematics*[5]. Indeed, today's scientific theories are codified by a mathematical formalism together with a minimal interpretation (which is usually operationalistic) for the theoretical terms linking these to appropriate elementary experiments. The method of science can thus be characterised by three moments: A mathematically codified theory, Experimental praxis, and a minimal interpretation connecting theory and experiment. In general this is sufficient for the *internal* discourse of natural sciences and its technical *applications*.

If we ask for the relation between the rigor of mathematical formalism and the ambiguity of the real world we might answer as Albert Einstein (1879-1955) put it:

> *As far as the propositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality*[6].

In spite of Albert Einstein's well-known ironical warning since the days of Galileo Galilei natural science uses mathematics as if it were the language of nature. Of course, Einstein's theory and philosophical attitude, is a paradigmatic example for this.

3.1 Mathematics as the language of natural science

The metaphor of mathematics being the language of nature is at least as old as modern science itself; it may be sufficient to recall Galileo Galilei's (1564-1642) famous quote from the *Il saggiatore* (The Assayer) where he states:

> *Philosophy [nature] is written in that great book which ever is always before our eyes – I mean the universe – but we cannot understand it if we do not first learn the language and grasp the characters in which it is written. It is written in the language of mathematics, and the characters are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth*[7].

This claim might be tough stuff for all non-mathematicians—who probably did not have the impression to be stumbling helplessly through a dark labyrinth.

[5] See I. Kant: "Metaphysische Anfangsgründe der Naturwissenschaft." A IX.

[6] See A. Einstein: *Geometrie und Erfahrung.* In: Ders.: "Mein Weltbild." Ullstein, Berlin 1988, S. 120.

[7] See G. Galilei: "Il saggiatore." p. 25.

It is, however, not nature itself, but the books of natural science which prove GALILEO's statement true. In fact, the universal language of these books *is* mathematics. Though the Phythagorean creed—GALILEO empasised so brilliantly—may be the conviction of many scientists, the equation 'mathematics=language of nature' has been questioned from both sides. On the one hand, philosophers emphasised that mathematics could only grasp *some* aspects of nature, that the whole reality of the world is much ritcher than any mathematical structure could grasp—and this holds already for Aristotle. On the other hand, in the 20th century mathematics became more and more independent from its linkage to the sciences. Only the character of language remainded from GALILEO's metaphor, but it is not obligatory that mathematics has to talk about nature. If you adopt a strictly formalistic view, mathematics is not obliged to talk about any object—it is then a language without any meaning. However, by giving up any semantical commitments mathematics gained an enormous flexiblity to define and examine various structures. As a result, out of this stock the demand of the scientists could be satisfied even easier.

It could thus seem as if mathematics were just a neutral language; only the content is relevant not the form. However, to use mathematics as the language of science has many material implications which should be analysed also critically. Without any claim of completeness I will now sketch some special features of the mathematical language. How far this language is suitable for a special situation must be decided case by case.

1. Mathematics is unique as a language by its extremely broad and at the same time extremely clear cut criterion for the ongoing or ending of the communication, respectively. Any 'false' proposition or (steps of) argument or calculation must be ruled out, however, *only* these. Mathematics can talk about *any* object, whose structure could grasped by by true or false propositions. The German philosopher and sociologist NIKLAS LUHMANN (1927-1998) characterised mathematics by a peculiar combination of indetermination with respect to content and determination with respect to form—similar to money only[8]. One effect of this is, that usually discussions among mathematicians about the validity of an argument are comparably short.

2. To obtain this rigor, mathematics has to rule out any vagueness of its symbols (as far as this is possible). Strict identity of the marks is asured by

[8]See N. LUHMANN: "Die Wissenschaft der Gesellschaft." Suhrkamp, Frankfurt 2009, p. 200.

definition—every x will remain just the same x throughout a mathematical paper without any dependence of the context. The translation of a mathematical text is nothing else than a mere change of notation and thus possible without any loss. This is in contrast to all other natural languages, where translation means always interpretation[9]. With mathematics the situation is much easier, the language is clear as crystal, but at the same time we lose the ability of languages to express ambiguities or less sharp passages in meaning, we loose, e.g., humor, irony, esprit, tact.

3. Moreover, the mathematical discourse has a double face of despoty and subversion. The **despotical** aspect is nicely illustrated in the following cartoon.

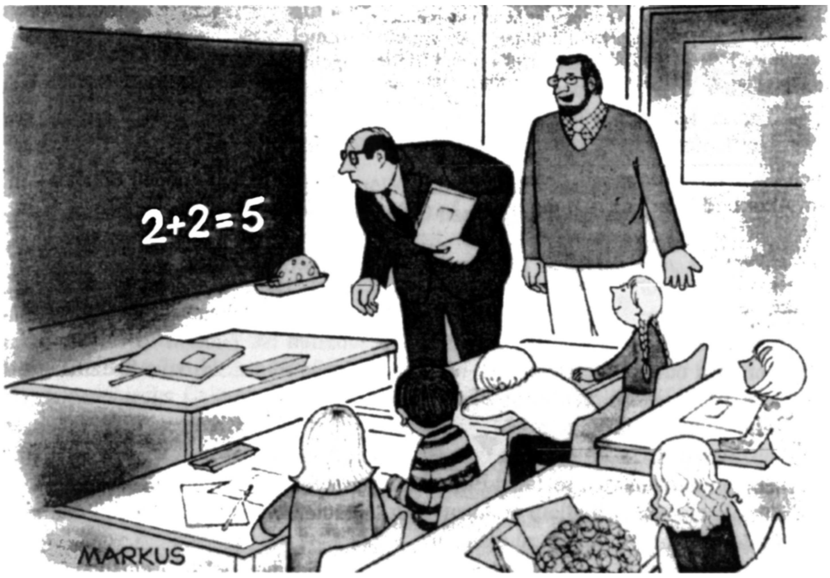

Figure 1: Sorry, principal, but we made a poll about the result.

Contrary to this classroom situation, there is no democracy in mathematics[10]. On the other hand, the social position of the dialogue partners is

[9]The effect of (bad) translations is quite dramatical if you compare sophisticated argumentations in your mother language with that kind of piggin, international science nowadays uses.

[10]The winning of dialogues is consequently the intuitive background for a constructive approach to formal logic by Paul Lorenzen and Oswald Schwemmer. A claim is provable, if you have a definite strategy for winning such a dialogue; see P. Lorenzen, O. Schwemmer: "Konstruktive Logik, Ethik und Wissenschaftstheorie." BI, Mannheim 1973. It is however, still an open question, *how* we *see* that such a strategy will necessarily lead to the goal.

completely irrelevant; to defend a mathematical theorem you cannot invoke authorities, only a coherent proof counts, the discours is 'herrschaftsfrei' (free of domination) in an ideal sense; thus there is also a **subversive** aspect.

Once such a (formal) proof is given every opposing view is without any chance. You cannot argue in favour of a mathematically disproved theorem without leaving the club of reasonable people. Then the proof exhibits is despotical aspect.

4. The impact on an object mathematical language has can be observed especially acurate when we look at the mathematisation of language itself, e.g., if we observe formalised or mathematical logic. Let me quote David Hilbert (1862-1943), the leading mathematician of the 20th century, who claims

   > *that the usage of language is the essential characteristics, by which human beings separate from all other creatures*[11].

   His reduction of language to its 'essence', however, reduces its function to mere unique denotation:

   > *If we survey the languages familiar to us, we observe a great similarity in structure. The differences are basically just the conventions to use different words, different names (...) It is completely irrelevant if you say 'table', 'mensa' or 'Tisch' and tree, Baum, arbre or dendron*[12].

   The result of his research for the basic structure of language[13] is formal logic, thus the

   > *articulation of thoughts becomes essentially the operating with notions*[14].

[11] "[...] dass dasjenige Hilfsmittel, durch das sich der Mensch über die anderen Lebewesen erhebt, im wesentlichen die Sprache ist." For this and the following quotes see D. Hilbert: "Wissen und mathematisches Denken." Vorlesung ausgearbeitet von Wilhelm Ackermann. Göttingen 1988, pp. 92.

[12] "Wenn wir die Sprachen, die uns nahe stehen, überblicken, so dringt sich die Ähnlichkeit in der Struktur auf. Die Unterschiede sind wesentlich nur die Konvention, dass andere Worte, andere Namen gebraucht werden. [...] Ob man table, mensa oder Tisch, [...] ob man tree, Baum, arbre oder dendron sagt, ist ja ganz unwesentlich und gleichgültig."

[13] "auf die Struktur der Sprache gerichteten Untersuchung"

[14] "das Aussprechen der Gedanken wird wesentlich zu einem Operieren mit Begriffen."

It is a special irony if he characterises the goal of formal logic by a mechanization of human reasoning. Paul Isaac Bernays (1888-1977) one of Hilberts most prominent pupils states it as follows:

> *After having found the principles of logical reasoning, nothing else has to be thought. The rules of reasoning must eliminate the logical* thinking. *Or else we would need other rules, how the first rules must be applied. This requirement of an expulsion of spirit could in fact be fulfilled*[15].

Probably essential aspects of natural language *and* of human reason were eliminated thereby. This is pinpointed by an ironical remark of Kant's contemporary Johann Georg Hamann (1730-1788). He criticised the monarchy of mathematics, the strange forcing character of its proofs, becoming the ideal of any reasoning. Should the uniqueness and inevitablity of the results be the essential advantage, then human reason would have found a much better working equivalent in the instict of an insect.

> *Endlich versteht es sich am Rande, daß, wenn die Mathematik sich einen Vorzug des Adels wegen ihrer allgemeinen und nothwendigen Zuverläßigkeit anmaaßen kann, auch die menschliche Vernunft selbst dem unfehlbaren u[nd] untrüglichen* Instinct *der Insecten nachstehen müßte*[16].

3.2 Why mathematics for the sciences?

But why does mathematics play this indispensable role for natural sciences? Again we could quote Kant:

> *They [all students of nature] learned that reason has insight only into that which it produces after a plan of its own (...). Reason, holding in one hand its principles, according to which alone concordant appearances can be admitted as equivalent to laws, and in the other hand the*

[15] "[N]achdem einmal die Prinzipien des Schließens genannt sind, [braucht nun] nichts mehr überlegt zu werden. Die Regeln des Schließens müssen so beschaffen sein, dass sie das logische *Denken* elimieren. Andernfalls müßten wir ja erst wieder logische Regeln dafür haben, wie jene Regeln anzuwenden sind. Dieser Forderung der Austreibung des Geistes kann nun wirklich genügt werden." See P. Bernays: "Abhandlungen zur Philosophie der Mathematik." Wiss. Buchgesellschaft, Darmstadt 1976, p. 9.

[16] See O. Bayer: "Hamanns Metakritik Kants." frommann-holzboog, Stuttgart 2002, p. 296.

experiment which it has devised in conformity with these principles, must approach nature in order to be taught by it. It must not, however, do so in the character of a pupil who listens to everything that the teacher chooses to say, but of an appointed judge who compels the witnesses to answer questions which he has himself formulated. Even physics, therefore, owes the beneficent revolution in its point of view entirely to the happy thought, that while reason must seek in nature, not fictitiously ascribe to it, whatever as not being knowable through reason's own resources has to be learnt, if learnt at all, only from nature, it must adopt as its guide, in so seeking, that which it has itself put into nature[17].

The mentioned principles of reason are given by mathematics; KANTs argument for this, briefly, is the following: Any empirical science needs a theoretical framework first. This theory must be *a priori*, thus it cannot be based on the reality but mere possibility of objects, since reality could only be analysed by experience. The possible objects of the natural sciences, however, must be given in time and space—it is not sufficient, that the respective concepts are free of contradictions. Thus the theoretical concepts are to be based on *a priori* constructions in time and space—this, however, characterises the working of mathematics[18]. We will not go into a detailed discussion of this claim—especially the synthetic character of mathematics, which is so important for KANTs point of view is still quite controversial—but instead focus on the active role, the scientific observer plays in his concept. Another quote may emphasise this point:

[T]he order and regularity in the appearances, which we entitle nature, we ourselves introduce. We could never find them in appearances, had we not ourselves, or the nature of our mind, originally set them there[19].

Again, this thesis exhibits an ambiguous use of the word *nature* which is indicative of a central problem. On the one hand, there is the unity of the (human) subject, whose orderly internal "nature" is capable of developing mathematics. The use of this instrument in turn guarantees (a description and the manipulation of) the orderliness of the external "nature." Any all-too easy identification of

[17]KrV, B XIII.

[18]Compare the argumentation in I. KANT: "Metaphysische Anfangsgründe der Naturwissenschaft." A IX.

[19]KrV, A123.

these two natures leads—as we have seen—to a naive monism of various types—be it materialistic or idealistic.

4 Reason's Nature—Reason creating mathematics

In this second part, we will invert the direction of the title question. How to characterise the natur of reason? I will not even try to sketch a picture of the various abilities of human mind; already reason—as a special facet—has a tremendous variety of aspects. I will just consider *one* quite special and quite strange ability, namely our ability to produce mathematics. Following KANTs traces—we will thus focus on mathematics bridging between the human intellectual constructions and the empirical data. From an epistemological point of view we will ask for the 'nature' of mathematical structures.

4.1 Mathematics and Freedom

In the PLATONIC description, mathematical objects or structures are somewhat 'outside' the mathematician, eternal entities to be studied or contemplated. It is then a quite special ability of the human mind's eye to 'see' these eternal forms. And remark that PLATON profitted in his argumentation decisively from this fact. Just recall the arguments in *Phaidon* and *Menon* against any sceptical or naturalistic docrine and finally leading to the central metaphors of the *anamnesis* and dialectics as the *second best navigation*.

The first major break, however, with this purely theoretical character of mathematics is due to a theologian, NIKOLAUS CUSANUS (1401-1464). According to NIKOLAUS the human mind is the creator of the objects of mathematics—parallel to GOd's creation of the world:

> *The human mind, which is an image of the Absolute Mind and which in a human fashion is free, posits, in its own concepts, delimitations for all things; for it is a mind that conceptually measures all things. In this conceptual way it imposes a delimitation on lines, which it makes to be long or short; and it imposes end-points on the lines, just as it chooses to. And the human mind first determines within itself whatever it proposes to do; and it is the delimitation of all its own works*[20].

[20]See *De venatione sapientiae* c. 27 (h XII n. 82, 13-17): *Mens enim humana, quae est imago*

This prominent theological topos, namely the free *creatio ex nihilo* of the multitude of things out of and due to the (trinitatrian) unity of the Creator can indeed uniquely be understood using the model of mathematics[21]: The unity of the human mind generates the diversity of mathematical structures like God is creating the real beings:

> *[T]here is a single infinite equality-of-being unto which I look when I draw different figures. Therefore, [by comparison], when the Creator creates all things, He creates all of them while He is turned toward Himself, because He is that Infinity which is Equality-of-being. It is the One infinite equality of being, on which I look, if I draw the various mathematical figures. Turning to Himself the creator creates everything, since he is the infinity which is the equality of being*[22].

Moreover, the mathematical knowledge is more rigid than any other, precisely because the mathematical structures are our own constructions[23].

Going one step further, we see that mathematical structures neither drop from heaven nor can they directly be found in nature. As the history shows, mathematical concepts are not invented as completed and unchangable objects, but are shaped and improved during the centuries. Probably HUSSERL's concept of *Limesgestalten* is more adequate than the never-never land's ideas of platonism.

For me the important point is, that mathematics is intimately connected with a special aspect of human freedom; the freedom to define and choose consitant rules and to freely obey these. During the course of the early 20th century we can observe a major change with respect to this question. It is wellknown that the result is a switch from external to almost purely internal reference leading to a far reaching autonomy of mathematics. It lies in the *free* choice of the mathematician, which special set of axioms he likes to start with[24]. No external

mentis absolutae, humaniter libera omnibus rebus in suo conceptu terminos ponit, quia mens mensurans notionaliter cuncta. Sic ponit terminum lineis, quas facit longas vel breves, et tot ponit punctales terminos in ipsis, sicut vult.

[21]We are thus confronted with the question how Gods unity expresses herself in the multitude of the world. The figure of NIKOLAUS for this is *complicatio*, enfolding, and *explicatio*, unfolding. And it can just be illustrated by the rational action of the mind doing mathematics.

[22]See *De Complementis Theologicis*, [Cu, p. 668]: *Una igitur infinita essendi aequalitas est ad quam respicio, quando diversas depingo figuras. Creator igitur dum omnia dreat ad se ipsum conversus omnia creat, quia ipse est infinitas illa, quae est essendi aequalitas.*

[23]For a more profound analysis of Cuanus we refer to [Ni04], [Ni05a], [Ni05b], [Ni05c].

[24]HERBERT MEHRTENS discusses the development of modern mathematics and the disputes during the so called foundational crisis under this aspect of creative freedom, see H.

object dictates a certain set. And it is hardly exaggerated when Georg Cantor (1845-1918) claims:

> *The essence of mathematics is freedom*[25].

However, this freedom is restricted in a twofold way we discussed already above. First, there is no freedom of interpretation and no context dependence of the terms. There is—so to speak—no hermeneutical problem in a mathematical text. This strong concept of identity enables and leads to the second restriction: the chosen axioms are not allowed to contain contradictions neither explicitely nor implicitely. The anxious emphasis on this consistency, is the prize we pay for the freedom of choice with respect to the axioms. Thus mathematics could be characterised as being the free enfolding of human mind strictly respecting the self-limitation of identity and consistency.

5 Circles of Reflection

Coming to the last part of my talk I have to admit, that I apparently commited the same crime I accused the naturalist: I repeatedly used the expression 'reasonable' and I tried to give you arguments to be followed by reason—but the very question remained unsolved, what this myterious *reason* actually might be.

In fact, we find a remarkable insight in these ridiculous circle already in the Greek philosophy; Platon pinpoints it almost at the end of his Theaitetos—where a similar question for the essence of knowledge is discussed:

> *But really, Theaetetus, our talk has been badly tainted with unclearness all along; for we have said over and over gain "we know" and "we do not know" and "we have knowledge" and "we have no knowledge," as if we could understand ach other, while we were still ignorant of knowledge; and at this very moment, if you please, we have again used the erms "be ignorant" and "understand," as though we had any right to use them if we are deprived of knowledge*[26].

Mehrtens: "Moderne Sprache Mathematik." Suhrkamp, Frankfurt 1990. Here David Hilbert—following Georg Cantor—stands for a progressive modernity against Luitzen Egbertus Jan Brouwer (1881-1966)—following Leopold Kronecker (1823-1891)— being the representative of reactionary anti-modernity which claims a necessary external reference for mathematics.

[25] "Das Wesen der Mathematik liegt in ihrer Freiheit." (translation by the author), quoted from W. Purkert, H. J. Ilgauds: "Georg Cantor." Birkhäuser, Basel 1987.

[26] Platon: Theaitetos 196e.

As it seems to be impossible to strictly avoid this sort of circulariry—or else to quit talking at all — in my concluding section I will now focus on this phenomenon.

It is often claimed that mathematics takes a special third position between the natural sciences and the humanities. I will now characterise this position with respect to the built in reflection. Thus, the criterion is the ability of a science to reflect about its own foundations by its own methods.

Natural sciences are ruling out such reflections on the foundations systematically. Of course, there *are* intensive discussions about the basic concepts of theory and experimentation especially before paradigm changes—see the change in time and space concepts due to special relativity or the change in the state concept due to quantum mechanics. However, even these revolutions remainded within the framework of mathematical theory and experimental praxis. Science itself remains completely outside its objects, the observer in physics is not its own problem. If these circles are thematised explicitely we run into paradoxes. The question of the essence of physics is not a physical question, that is, it will not be tackled by physical methods. In fact, it would be unfair to require an experiment by which we could answer the question, whether the experimental method is valid at all. Georg Picht (1913-1982), a German philosopher, expressed this position very clearly:

> *Natural scientists can do there research only, because since Galilei they decided to ignore the immensely difficult question what it is that enables their knowledge.They do not ask for nature in itself, since the became aware, that the renouncement to posing this question opens a wide playing ground for the naive research on phenomena* within *nature*[27].

In constrast to this, reflection is an integral feature of all humanities, and especially for philosophy. Philosophy and its method is a problem and an object for philosophy which must never be forgotten.

Concerning mathematics, we observe a strange phenomenon: The very foundations of mathematics could be discussed by mathematical methods, it could be

[27] "Die Naturwissenschaftler können ihre Forschungen nur deshalb betreiben, weil sie seit Galilei beschlossen haben, die unermeßlich schwierige Frage, was sie zu ihren Erkenntnissen befähigt, auszuklammern. Sie fragen nicht nach der Natur überhaupt, weil sie entdeckt haben, daß der Verzicht, diese Frage zu stellen, ihnen Spielraum gibt, sich unbefangen der Erforschung der Phänomene *innerhalb* der Natur zu widmen." See G. Picht: "Der Begriff der Natur und seine Geschichte." Klett-Cotta, Stuttgart 1990, p. 4.

formulated as a mathematical problem and—almost (!)—be solved. Of course, the most intensive phase of this endeavor was the foundational debate (or crisis) of the 20th century's first half. Here, the primarily philosophical question for the laws of reasoning, the acceptable methods of proof and the certitude of mathematical theorems could be translated into mathematical or 'meta-mathematical' questions; the concerned mathematical subdisciplines were: Mathematical logic, Proof theory, set and model theory. To briefly characterise this approach, the *process* of a mathematical proof could be translated into a formal series of signs and thus into a mathematical *object*. So one could formalise provability and show mathematical theorems *about* this mathematical reasoning. Finally, these foundational work became just another mathematical subdiscipline and the working mathematician could continue to do his job unburdened. Certainly, the results of Kurt Gödel showed heavy restrictions to this foundational approach, however, by mathematical means!

Thus, mathematics did not completely ignore the 'immensely difficult question what it is that enables its knowledge', but it could mitigate it in a way such that it will not bother the ongoing of the research. There is no formal, thus no strict proof for the soundness of the (or any sufficiently powerful) axiomatic ground of mathematics, just intuitive arguments. However, these are sufficient to go on with the work without any further counterinsurance. If there appears any contradiction within a branch of mathematics it will be ruled out radically; but it serves also as a wellcome impulse to meliorate the foundations. Examples for this are the deepening of the concept of a function—from Leibniz to Cauchy and Weierstrass—and the developement of set theory after Bolzano and Cantor.

In the last consequence mathematics can only be analysed by mathematical methods if one accepts contradictions, and that means not at all. However, this at least can be shown by mathematical reasoning. Let me finally quote Bernays again:

> *A philosophical interpretation of the antinomies of axiomatic set theory is that mathematics as a whole is not a mathematical object. Thus, mathematics can only be understood as being an open plurality*[28].

[28] "Philosophisch kann das Verfahren der Lösung der Antinomien durch die axiomatische Mengenlehre in dem Sinne gedeutet werden, [...] dass man die Antinomien als Anzeichen dafür nimmt, dass die Mathematik als Ganzes nicht ein mathematisches Objekt bildet und dass also die Mathematik nur als eine offene Mannigfaltigkeit verstanden werden kann." See P. Bernays: Abhandlungen, p. 174.

I hope I could show that it is worth the trouble to examine this open plurality— called mathematics— more closely,

1. since it encourages reason against any form of naturalism (and naive monism),
2. since it shows a strange form of relatedness of human reason to freedom,
3. and, finally, since it gives a model for the strange phenomenon of reflection.

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